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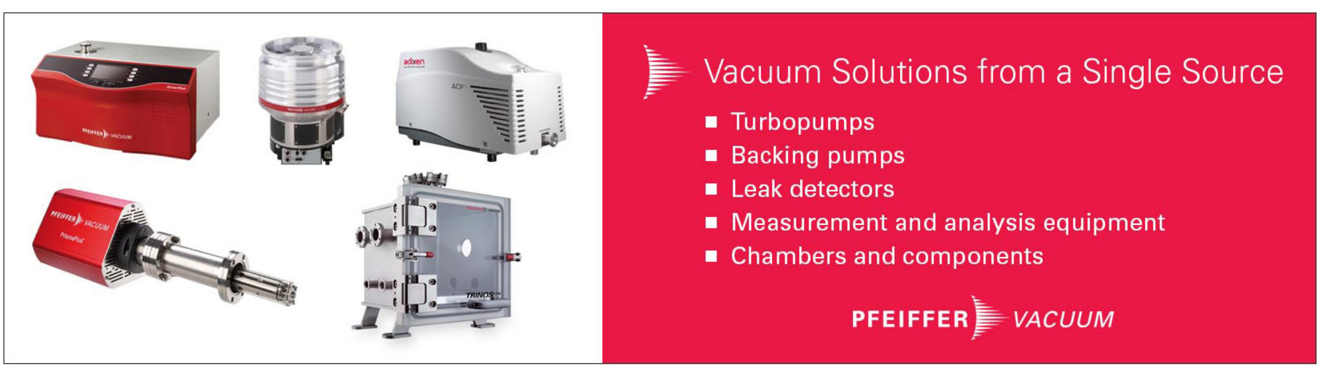

A new flow regime in a Taylor–Couette flow

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In this Brief Communication, we report a new finding on a Taylor–Couette flow in which the outer cylinder is stationary and the inner cylinder is accelerated linearly from rest to a desired speed. The results show that when the acceleration ($d\mathrm{Re}/dt$) is higher than a critical value of about 2.2 s^{-1}, there exists a new flow regime in which the flow pattern shows remarkable resemblance to regular Taylor vortex flow but is of shorter wavelength. However, when the acceleration is lower than 2.2 s^{-1}, a wavy flow is found to occur for the same Reynolds number range. To our knowledge, this is probably the first time that such a phenomenon has been observed. For completeness, the case of a decelerating cylinder is also investigated, and the results are found to be almost the same. [S1070-6631(98)01412-3]

Since Taylor's[1] famous publication in 1923 on the stability of a circular Couette flow between rotating cylinders, a great deal of theoretical and experimental work has been published on the flow stability between two concentric cylinders (see, e.g., Refs. 2–12). In 1965, Coles[6] was the first to report the nonuniqueness of the wavy flow in the Taylor–Couette flow. He noted that for a given Reynolds number, as many as 20–25 different states (characterized by axial wavelength and azimuthal wave number) were seen, and these states were found to be a function of the initial conditions as well as the manner in which the inner cylinder was accelerated to the final speed. Although numerous studies (see Ahlers *et al.*,[2] Andereck *et al.*,[3] Coles,[6] Park *et al.*,[10] and Burkhalter and Koschmieder[11,12]) have shown the importance of acceleration/deceleration in determining the final state of the flow, these investigations were somewhat limited in scopes. For example, in Coles[6] and Ahlers *et al.*,[2] no quantitative values of the acceleration/deceleration used in their investigations were given. Therefore, the path to which the flow is subjected is not accurately known. On the other hand, although Park *et al.*[10] had provided quantitative values of the acceleration, his investigation was limited only to the onset of Taylor vortex flow (TVF). Similarly, in the investigation by Burkhalter and Koschmieder[11,12] on the axial wavelength of supercritical axisymmetric Taylor vortex flow, the accelerations used were restricted to only two extreme conditions (i.e., ''sudden start'' and ''quasi-steady''). The lack of systematic study of the effect of acceleration/deceleration on the Taylor–Couette flow motivated us to carry out the present investigation. Here, our attention is focused primarily on the case of linear acceleration/deceleration ($\pm d\mathrm{Re}/dt$) that ranges from 0.02 to 200 s^{-1}, and the Reynolds number (Re) varying from the onset of Taylor vortex flow (Re_c) to about 5 Re_c. To our knowledge, this is probably the first time that such a broad range of accurately known accelerations have been used.

The experiments were carried out in an apparatus consisting of an inner aluminum cylinder with outer radius of $R_1 = 75.5$ mm, and a stationary outer precision perspex cylinder with inner radius of $R_2 = 94.0$ mm. This gives the radius ratio of $\eta = 0.8032$. The height of the fluid between the two cylinders is 0.935 m, resulting in the aspect ratio $\Gamma = 50.54$. The Reynolds number is defined as $\mathrm{Re} = R_1 \Omega d/\nu$, where Ω is the angular frequency of the inner cylinder, d is the gap size between two cylinders, and ν is the kinematic viscosity. The acceleration is defined as the rate of change of Reynolds number: $a = d\mathrm{Re}/dt = (R_1 d/\nu) d\Omega/dt$. The working fluid is a mixture of 66% glycerin and 34% water with the kinematic viscosity of 10.516 cS at the room temperature 27 °C. To visualize the flow, Kalliroscope AQ-100 reflective flakes were added to the solution, and the resulting motions were monitored and captured with the aid of a 5 W argon ion laser and a charge-coupled device video camera.

In this study, the motion of the inner cylinder is controlled by a PC through a microstepper motor. Unless otherwise, stated, the inner cylinder is accelerated (or decelerated) linearly from rest (or from a given initial speed) to a predetermined speed, and the time interval between the initial and the final speed is adjusted to achieve the acceleration required. With the present setup, the maximum acceleration/deceleration which could be achieved is $d\mathrm{Re}/dt \cong \pm 200\ \mathrm{s}^{-1}$, and the minimum acceleration/deceleration is $d\mathrm{Re}/dt = \pm 0.02\ \mathrm{s}^{-1}$ (i.e., quasi-steady condition). Note, however, that once the cylinder has reached the final speed, it is maintained at the same speed for the rest of the experiment which may last as long as 12 h.

Figure 1 is a chart summarizing the ''history effect'' of acceleration on the state of the Taylor–Couette flow. Each point in the chart represents an experimental data taken at least 2 h after the inner cylinder has reached the final speed. Figure 1 clearly shows that when the acceleration (dRe/dt) is less than a critical value of 2.2 s^{-1}, the state of the flow when the Reynolds number increases follows a well-known sequence of circular Couette flow, followed by regular Taylor vortex flow and finally by wavy vortex flow. The first sign of the wavy mode in this experiment occurs at $\mathrm{Re}/\mathrm{Re}_c$

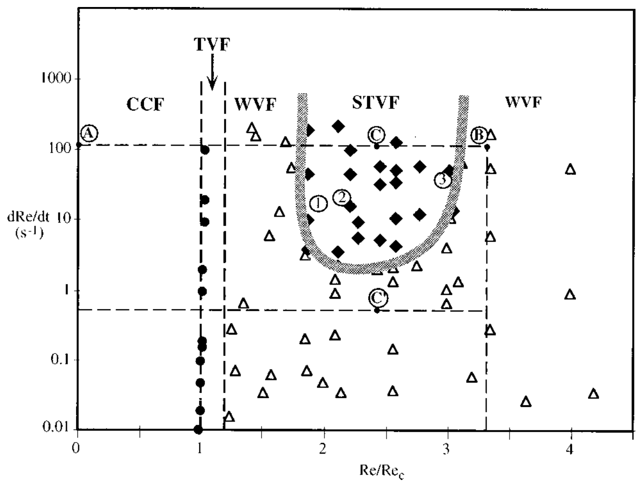

FIG. 1. A chart showing the effect of acceleration on the state of a Taylor–Couette Flow. CCF: Circular Couette flow; TVF: first Taylor vortex flow; STVF: Second Taylor vortex flow; WVF: wavy vortex flow. The shaded curve indicates the approximate transition boundary between the wavy vortex flow and the second Taylor vortex flow. ●—First Taylor vortex flow; △—Wavy vortex flow; ♦—Second Taylor vortex flow.

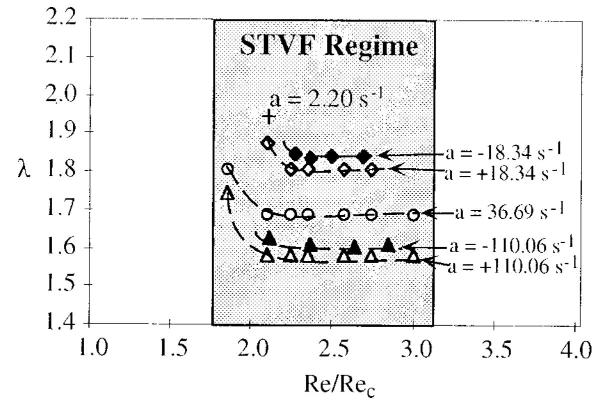

FIG. 2. Axial wavelength of STVF vs Re/Re_c for different acceleration/deceleration ($dRe/dt = \pm a$). For the purpose of comparison, only two extreme values of deceleration are presented. Note that the λ for the first TVF is approximately 2.

≈ 1.15. Here, the flow pattern appears as a slight rocking of the boundaries between adjacent vortex cells, in a fashion which could be accounted for by the presence of a single traveling circumferential wave occupying the entire circumference. This rocking behavior is found to occur only within a very narrow range of Re. When $Re/Re_c \approx 1.2$, the number of azimuthal waves is increased to two (or $m=2$ mode), and beyond $Re/Re_c \approx 1.8$, $m=3$ mode is observed and remains throughout the entire range of Reynolds numbers tested.

However, it can be seen in Fig. 1 that when $dRe/dt > 2.2\ s^{-1}$, there exists a new flow regime which to our knowledge has not been seen before. In this regime, which is bounded by a parabolic-shaped curve (see shaded curve in Fig. 1), the flow pattern is remarkably similar to the Taylor vortex flow (TVF) normally seen at Re_c. Here, the vortices are stable and regular, except that their average axial wavelength is much shorter than that in TVF. Moreover, the flow occurs in the Reynolds number range where the wavy vortex flow would normally occur. To ascertain that the vortices have stabilized after 2 h, a few points (indicated by 1, 2, and 3 in Fig. 1) were selected for observation 8–12 h after the final speed is reached, and the results were found to be the same as for the case of 2 h. Because of its similarity to the TVF, and also for ease of reference, we shall refer to the new flow as a second Taylor vortex flow (STVF). However, it should be stressed that the formation of Taylor vortex flow at Reynolds number higher than Re_c has been observed previously by Burkhalter and Koschmieder.[11,12] In fact they reported that for a radius ratio (η) of less than 0.727, Taylor vortex flow is found to exist up to 9 Re_c for sudden start and quasi-steady conditions. They referred to the flow as supercritical axisymmetric Taylor vortex flow. However, they did not report seeing the same flow phenomenon depicted in Fig. 1. Moreover, the STVF observed in the present study ($\eta = 0.8032$) occupies a narrower range of Reynolds number, and occurs only when the acceleration is higher than 2.2 s^{-1}. Although STVF may well be the same as the supercritical Taylor vortex flow reported by Burkhalter *et al.*,[11,12] it is not possible to make any meaningful comparison because our setup is quite different from theirs, particularly in the radius ratio which has previously been shown to affect the stability of the flow.

As mentioned earlier, the averaged axial wavelength (λ) for the second Taylor vortex flow is found to be much shorter than that for the first Taylor Vortex flow. Here λ is defined as $2H/Nd$, where H is the height of the fluid column, and N is the number of axial vortices. The above observation led one to suspect that the vortices in the new flow regime may be a function of both the Reynolds number and the ''history effect'' of acceleration. This has led us to carry out a thorough investigation, which confirms our suspicion, and the results are clearly displayed in Fig. 2 for the STVF case only. Here, it can be seen that for a given acceleration, the axial wavelength of the vortices at first decreases sharply and then approaches a constant value as the Reynolds number increases. The sharp drop in the wavelength is found to occur near the left-hand boundary of the second Taylor vortex stability curve. Similarly, one can deduce from Fig. 2 that for a given Reynolds number, decreasing the acceleration would lead to an increase in the axial wavelength.

For completeness, we also examined the ''history effect'' of acceleration on the wavelength of the first Taylor vortex flow, and the result shows that irrespective of acceleration, the average axial wavelength remains relatively constant at about 2.0. In addition, we also carried out investigations on the effect of deceleration on the stability of the flow, and similar results are obtained. More specifically, if the inner cylinder is first accelerated from rest to a high Reynolds number flow regime, say $Re/Re_c = 3.4$ for example (i.e., path $A \rightarrow B$ in Fig. 1). And if after 2 h of delay, the cylinder is decelerated to a lower Reynolds number along the path $B \rightarrow C$, the flow pattern at C is found to be STVF, except that the axial wavelength is slightly larger than that of the accelerating case (see Fig. 2 for comparison). On the other hand, if the speed of the cylinder is increased along the path $A \rightarrow B$, and after 2 h, it is reduced to the condition at C', with

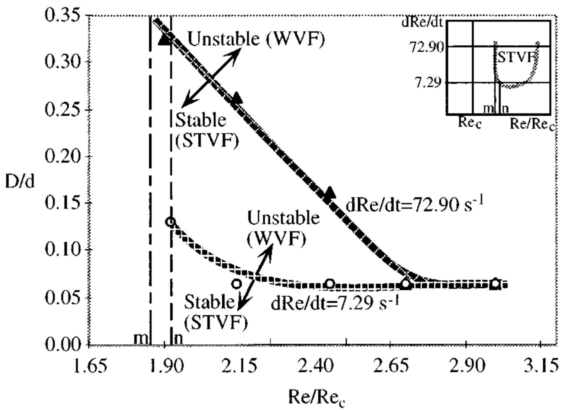

FIG. 3. A chart showing the right-hand boundary between STVF and WVF for different Reynolds number and rod size. ○—$d\mathrm{Re}/dt = 7.29\ \mathrm{s}^{-1}$; △—$d\mathrm{Re}/dt = 72.90\ \mathrm{s}^{-1}$. D = diameter of cylindrical rod, d = gap size between the cylinders. The insert on the top right-hand corner depicts a schematic drawing of Fig. 1. Here, the labels ''*m*'' and ''*n*'' correspond to the ones shown.

a constant deceleration of less than 2.2 s^{-1}, the flow under this condition is found to remain wavy.

To explore the stability of STVF, we decided to perturb the flow with cylindrical rods of different sizes and see how the flow responds to such disturbances. The procedure involves introducing a rod into the flow and letting it remain inside for one revolution of inner cylinder rotation before it is removed. Figure 3 shows the results for the case of $d\mathrm{Re}/dt = 72.90\ \mathrm{s}^{-1}$ and $d\mathrm{Re}/dt = 7.29\ \mathrm{s}^{-1}$. It can be seen from Fig. 3 that for a given Reynolds number and acceleration, there is a critical rod size, below which the disturbance introduced would eventually die down, and the flow returns to the stable condition. On the other hand, too large a disturbance would lead to the STVF developing into a wavy vortex flow. To ensure that the wavy flow observed is not in a state of transition, some points were selected for observation 8 h after the disturbance has been removed, and the results show that even after 8 h, the wavy flow remains wavy and there is no sign of it returning to the stable mode of STVF. Another way of interpreting the above results is to have them plotted in terms of the parameters shown in Fig. 1. A point of particular interest is the right-hand boundary between the STVF and the wavy flow (i.e., shaded area). It can be deduced from the results that for a given acceleration, say $a = 72.9\ \mathrm{s}^{-1}$, increasing the disturbance (or rod size) would result in the the right-hand shaded boundary shifting toward the left-hand side of the chart. In other words, the larger the disturbance the smaller is the region of STVF. This suggests that if the disturbance is large enough, the STVF regime may disappear completely. To test this hypothesis, a cylindrical rod of diameter $D/d = 0.43$ is introduced into the STVF flow with $a = 72.9\ \mathrm{s}^{-1}$ and $\mathrm{Re}/\mathrm{Re}_c = 1.87$. It is found that 4 h after the rod was removed, the flow still remained wavy and there was no sign of it returning to the STVF regime. For the case of low acceleration (i.e., $a = 7.29\ \mathrm{s}^{-1}$), a similar behavior is also found to occur, except that the stability of the flow is more sensitive to the applied disturbance as can be easily deduced from Fig. 3. From the above discussion, it may be postulated that, because of the flow sensitivity to small disturbance, the contour of the shaded boundary at low acceleration range (see Fig. 1) is likely to be different in another apparatus which may have a different inherent disturbance.

To conclude, we would like to emphasize that this investigation is more than just a nonuniqueness of the Taylor–Couette flow, which has been known for more than 30 years. An important outcome of this investigation is the discovery of a new flow regime, which to our knowledge has not been seen before. This finding is significant because it implies that it is possible for two distinct flow states (either STVF or wavy flow) to coexist at the same Reynolds number, with acceleration/deceleration as a controlling factor.

ACKNOWLEDGMENTS

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